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Specification and Design of Steel Gusset-Plates

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GUSSET-PLATESBY T. H. RUST,¹ Assoc. M. Am. Soc. C. E.

SYNOPSIS

An attempt is made in this paper to supply information about the structural steel plates designed to transfer force from one member to another in a structural steel frame or bridge. The forces enter and emerge from these gusset-plates, often by means of rivets, but sometimes through welds. Since a gusset-plate is a point of discontinuity of stress transfer in a structural steel frame, it is here that failures most often occur.

As early as 1867, J. W. Schwedler noted ² the unequal distribution of stress among the rivets in a riveted connection. He found that the end rivets took most of the stress and the interior ones least. Likewise, the ends of a welded connection are more highly stressed than the interior.

This inequality of stress transfer into a gusset-plate can be shown by stressing models of the type shown in Fig. 1, which are carved from single pieces of bakelite to simulate gusset-plates. When forces are applied through the legs to the plate, fringed photographs can be made by the aid of photo-elasticity. The photo-elastic fringes are the means of establishing the stress value $(p - q)$, at a particular point, p being one principal stress and q the other principal stress at right angles to it. Although a $(p - q)$ -determination is a good indication of the condition of stress in a plate, a $(p + q)$ -determination was made for some of the models. The lateral expansion or contraction of the stressed plate at any point is proportional to the $(p + q)$ -stress at that point. This change in thickness was measured by a light interferometer. From the $(p - q)$ and the $(p + q)$ -stress determinations, the value of the principal stresses, p and q , can be found.

Studies of several different shapes are reported herein and the stresses encountered are discussed. The models were designed with special reference to

NOTE.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **January, 1939, Proceedings.**

¹ Chicago, Ill.

² "Ueber Vernietungen," von J. W. Schwedler, *Deutsche Bauzeitung*, Architekten-Vereins zu Berlin I, November 29, 1867, No. 48, S 463-465.

gusset-plates. A part of the paper is devoted to recommendations for the design and specification of steel gusset-plates.

TESTS

Tensile forces were applied to the model shown in Fig. 1 by means of steel pins through the bushings. A force enters through the top pin, passes down the upper legs into the plate and is resisted by a reaction passing through the lower legs to the other pin, which is held in a stationary position. As shown in the side view of Fig. 1, the bushings are beveled to keep the force, P , on the center line of the model. At all junctions of the legs and the plate small fillets were provided to minimize local stress concentration. The plates were each about 0.2 in. thick, and the legs were attached to the plate for a distance of 0.75 in.

To avoid residual stresses in the edges of the bakelite plate, the models were annealed and polished as rapidly as possible. They were then subjected to forces, P , in the path of polarized light (see Fig. 2). A theory is that polarized

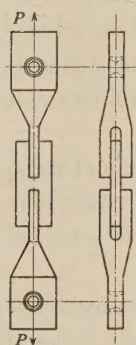


FIG. 1

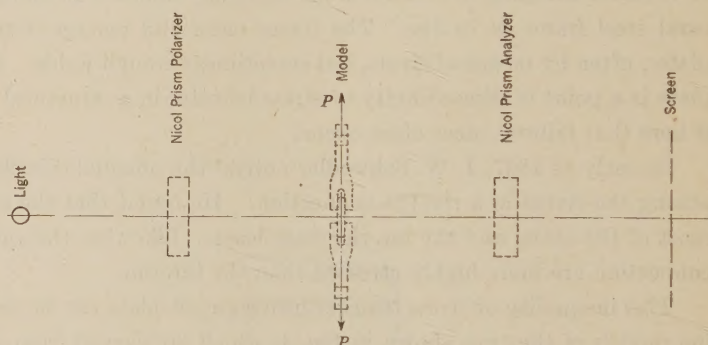


FIG. 2

light has its vibrations restricted to one plane, in contrast with ordinary light, in which light vibrations are said to be in many planes. When a beam of ordinary light is passed through a Nicol prism the light vibrations are held to a single plane, and it is then known as polarized light. If this light beam is passed through a second Nicol prism with its axis at right angles to the first prism, the beam disappears, and the field becomes dark. Since there are no vibrations perpendicular to one plane in a beam of interrupted polarized light, the second prism (termed the "analyzer") cannot transmit any light because its axis is at right angles to the axis of the first Nicol prism or polarizer.

If a polished bakelite model under stress is placed between the two prisms in the path of the polarized light, the second Nicol prism, or analyzer, transmits light between dark fringes. All the points on a particular dark fringe are of equal $(p - q)$ -stress in the plate of a model. Monochromatic mercury light was used.

The change in $(p - q)$ -stress from one fringe to another is constant for a given thickness of plate. This stress change between fringes is determined by subjecting a pilot specimen of the bakelite to known loads.

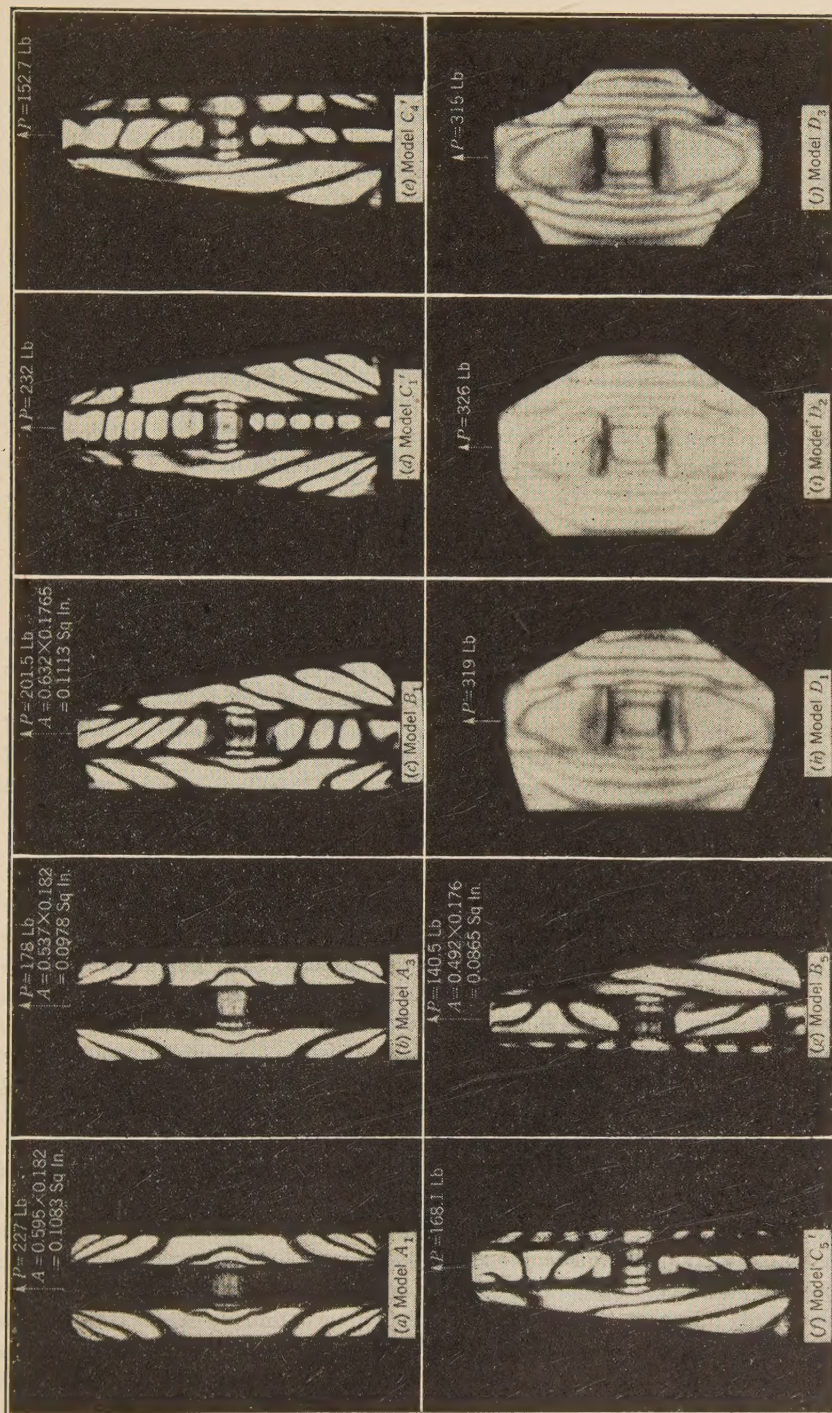


FIG. 3.—FRINGED PHOTOGRAPHS OF PLATES OF DIFFERENT SHAPES

Fig. 3 shows fringed photographs for plates of different shapes. The plates of Models A_1 , B_1 , and C_1' were $\frac{5}{8}$ in. wide at the mid-length; for Models D_1 , D_2 , and D_3 , the plates were 1.25 in. wide at the mid-length. For Models A_1 , B_1 , C_1' , D_1 , D_2 , and D_3 , the area of the plate at mid-length was nearly equal to the area of the two legs. The closeness of the fringes near the ends of the plates, at the legs, shows the rapid transfer of stress into the plates at those points. The unit stress in the legs is large at this point, whereas in the ends of the plate at the legs the unit stress is small. The difference in the unit stress of the legs and the plate causes rapid stress transfer. At the mid-length of the legs, the bakelite in the legs is stressed to about the same value as the plate. The comparatively great distance between the fringes in the plate at the mid-length of the legs, shows the less rapid transfer of stress at this point. Near the ends of the legs the unit stress is small in the legs; whereas the unit stress and unit deformation in the plates are large. This change in deformation accounts again for a rapid transfer of stress into the plate. The shear stress between the plate and the legs can be represented by a curve shown in Fig. 4. A smooth curve as indicated by the full line would be shown by a welded connection or a continuous connection such as exists in the models. A line of rivets would indicate a stepped curve as shown by the dotted lines in Fig. 4. A rivet or a row of rivets perpendicular to the length of the member would transfer an increment of shear into the plate equal to the area under the corresponding step.

If the cross-sectional area of the legs is equal to the area of the plate, the shear stress transfer at the end of the plate and at the ends of the legs would be of the same value. On the other hand if the area of the legs is less than the area of the plate there will be a greater transfer of stress into the plate at the end of the plate because of its greater stiffness, with correspondingly less transfer of stress at the ends of the legs, as shown in Fig. 5. If the legs were larger in area than the plate, the end ordinates of the curve would be reversed, as shown by the dotted curve.

Boundary Stresses.—At the outside boundary of a plate, symmetrical about the legs (as shown by Models A_1 , B_1 , and C_1' , Fig. 3), the change of stress is less rapid than it is next to the legs. If this were not so, the $(p - q)$ -fringes would extend from the sides of the legs, straight across the plate, to the boundary. The extension of the fringes, forward toward the mid-length of the plate, shows how stress lags at the boundaries of the plate. Corners of the plate are points of least accessibility of stress and the positions of zero stress. These corners, therefore, are good places to begin counting the fringes; each succeeding fringe represents a constant increase in $(p - q)$ -stress over the preceding fringe.

The boundaries at the mid-length of the plate are the most highly stressed. The stress at that point also lags considerably behind the stress in the center of the plate at the corresponding positions. At the mid-length of the legs in Model A_1 , Fig. 3 (which is approximately at the quarter-point of the plate), the fringes are nearly perpendicular to the boundary. There is comparatively little transfer of stress into the plate from the legs at this point, with less disposition of the stress to lag at the boundary. The same boundary stress condition is shown in Model C_1' , Fig. 3, at the third fringe from the top, and again less definitely at the third fringe from the bottom. The wide fringe on the wide

side of Model C_4' at the position of the upper leg shows that the increase in area of the plate practically compensates for the transfer of stress from the legs, causing little change in $(p - q)$ -stress in the plate at this point on the boundary. The third fringe from the top on the narrow side of the plate turns back. This is caused by the bending moment due to the eccentricity of the plate. The bending stress more than compensates for the tendency of the stress to lag at the boundary. The fourth fringe of Model C_4' turns forward again because the more rapid transfer of stress from the legs near the ends causes the boundary stress to lag again. The same phenomena are shown in Models C_5' , A_3 , and B_5 .

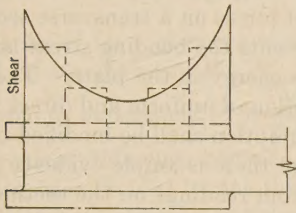


FIG. 4

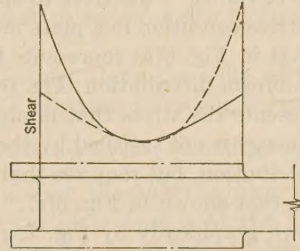


FIG. 5

In order to show something about the plate corner effects, the corners on the left side of Model D_1 , Fig. 3, were filed off for one-fourth the distance of the leg for Model D_2 . There is a more rapid change of boundary $(p - q)$ -stress near the left side of the leg in the plate of Model D_1 than in Model D_2 . Due to greater resistance to stress transfer in Model D_1 than in Model D_2 , the stress builds up more rapidly near the leg in Model D_1 than in Model D_2 . The upper left-hand corner of Model D_1 has a larger area free of stress than Model D_2 . Model D_1 has less continuity of stress transfer than Model D_2 in the upper left boundary. The $(p - q)$ -stress in the right side of the gusset-plate shows almost no change from the removal of the corners on the left side. The upper outside corner of the right side of the plate is at the end of the leg, whereas the lower outside corner is only three-fourths the way along the leg toward its end. The $(p - q)$ -fringes do not show any appreciable lack of symmetry due to this difference. The second fringe from the right on the right side of the plate bends out sharply near the boundary. This is true of the second fringe of all the D -models; for Model D_3 , Fig. 3, the third fringe also bends out sharply at the boundary. The rapidly increasing area of the plate is more than sufficient to counteract the addition of stress from the legs at these points. It should be noted that the stress transfer from the legs is a minimum near the mid-length of the legs.

Model D_3 , Fig. 3, was made by filing concave cuts in Model D_2 . In these models there is less unit shear on the right side of the legs near the end of the plate than on the left side of the legs, because there is less material there and, therefore, less resistance. The smaller unit shear causes less shear stress in the end rivets or ends of welds, as well as less bearing stress on the plate from the end rivets. Since the rivets at the end of a gusset-plate as well as their bearing stresses on the gusset-plate are critical, it is well to make the plate compara-

tively elastic by not having its surface area much larger than the end rivets require.

The vertical boundaries of the plate of the *D*-models are almost free of stress. These boundaries are so far from the direct path of stress transfer that they are not highly stressed, in accordance with the theory of least energy.

Interior Plate Stresses.—The greatest stresses in the interior of the plate are always between the ends of the legs of the models. All the stress has been transferred into the plate in this region, and it tends to concentrate in the most direct path of transfer. Some of the fringes curve out, which shows the tendency of the $(p - q)$ -stress to spread over a larger part of the plate.

The stress condition in a plate may be shown graphically by the diagrams in Fig. 6, that is, Fig. 6(a) represents the direct stress on a transverse section assuming uniform distribution; Fig. 6(b) represents the bending stress; and, Fig. 6(c) represents the stress that minimizes the energy in the plate. The theory of least energy is not satisfied by the assumption of uniform and direct bending stress distribution, but requires that the assumption shall be modified by stress similar to that shown in Fig. 6(c). Although there is ample evidence in these tests shown graphically by Fig. 7, plotted from readings on the section at the mid-length of the plate of Model A_1 (Fig. 3), there is no generalized theory for the least energy stress condition for Fig. 6(c). If the outside forces are applied

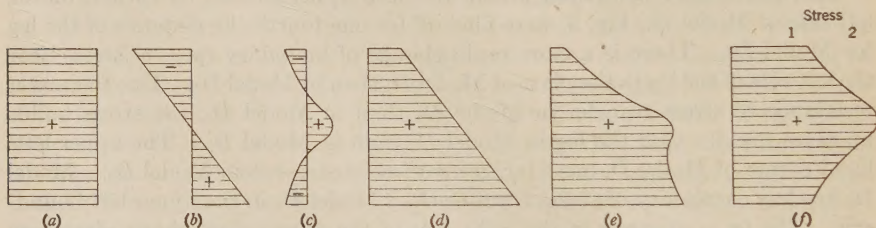


FIG. 6

FIG. 7

somewhere near the neutral axis of the plate this added least-energy stress will always have the general shape of the curve in Fig. 6(c), the negative area being equal to the positive area. Figs. 6(d) and 6(e) will have the area of Fig. 6(a), because equilibrium of the free body is always maintained between external and internal forces.

In nearly all the models, the fringes that pass near the ends of the legs change curvature. Since the largest concentration of $(p - q)$ -stress occurs in the plate between the legs, this stress tends to distribute itself over a greater part of the plate with the resulting change of curvature of these fringes.

With an eccentric plate such as that shown in Model C_5' the fringes denoting the highest $(p - q)$ -stress shift over toward the narrower side of the plate. The direct stress, the bending stress due to eccentric loading, and the stress due to the least energy in the plate, create a maximum stress to the right of the center line of the legs. In Model D_3 , Fig. 3, there is also a slight shifting of the fringes of highest value to the right of the center line of the legs. On the right side of this model there is less plate in the gusset corners than on the left side of the legs; therefore, the maximum stress shifts again slightly to the right.

In Model C_5' the almost horizontal fringes on the narrow side of the plate show that the bending stress due to eccentric loading is approximately equal to the decrease in least energy stress there. The principal stress, q , is not large except in the mid-length of the plate; even there it is considerably less than the p stress, as will be shown under the heading, "Discussion of Curves."

DISCUSSION OF CURVES

In addition to knowing something of the $(p - q)$ -stress distribution from the photo-elastic photographs of the models, it was desirable to know something also about stress distribution in gusset-plates from the plotting of stress curves.

Graphs of $(p - q)$ -stress and $(p + q)$ -stress along the sides of the legs of Model A_1 are shown in Fig. 8. The $(p - q)$ -stress difference between fringes was found by experiment to be $\frac{84.6}{0.183} = 470$ lb per sq in.; 84.6 lb per sq in. would be the stress difference between fringes in case the bakelite were 1 in. thick. This constant divided by the thickness of the plate used in Model A_1 gives the value of $(p - q)$ -stress difference between fringes. The positions of the $(p - q)$ -fringes together with their successive values were plotted for Model A_1 in Fig. 8.

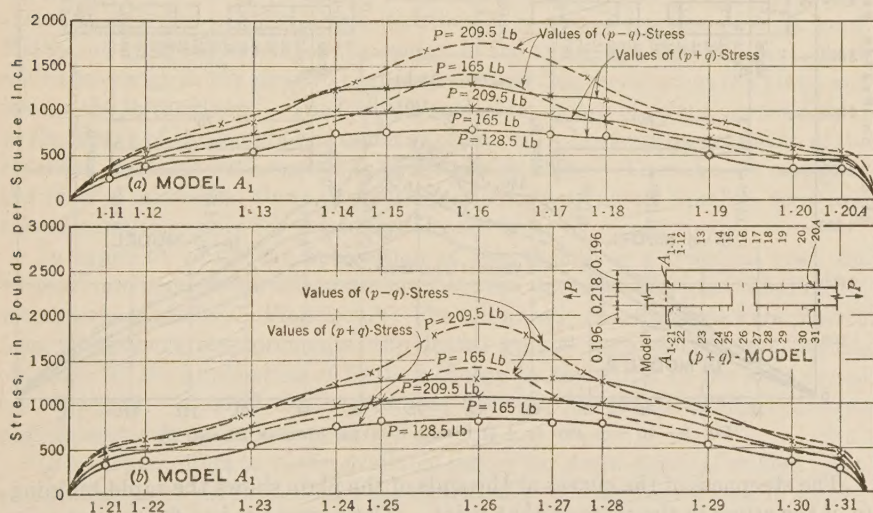


FIG. 8.— $(p - q)$ AND $(p + q)$ STRESS CURVES, MODEL A_1

The $(p + q)$ -curves were plotted from values obtained by measuring the lateral contraction of the stressed plates at various points along the legs. The lateral contraction or expansion at a point of the stressed plates is proportional to the $(p + q)$ -stress at that point. This lateral change in thickness of the bakelite plate was measured by a light interferometer,³ and the resulting stresses

³ "An Application of the Interferometer Strain Gage to Photoelasticity," by R. W. Vose, Jun. Am. Soc. C. E., *Journal of Applied Mechanics*, September, 1935, pp. A 99-102.

were plotted as shown. The $(p + q)$ -curves and the $(p - q)$ -curves nearly agree except at the mid-length of the plate, which indicates that there is little q -stress except at the mid-length of the plate. Since there is a large concentration of stress directly between the ends of the legs, an appreciable q -stress arises in the effort to distribute the stress over a larger part of the plate. The difference in ordinates between the $(p + q)$ -stress and the $(p - q)$ -stress curves for a particular load, divided by 2, gives the q -stress for a particular point. The addition of the ordinates, divided by 2, gives the p -stress at a particular point.

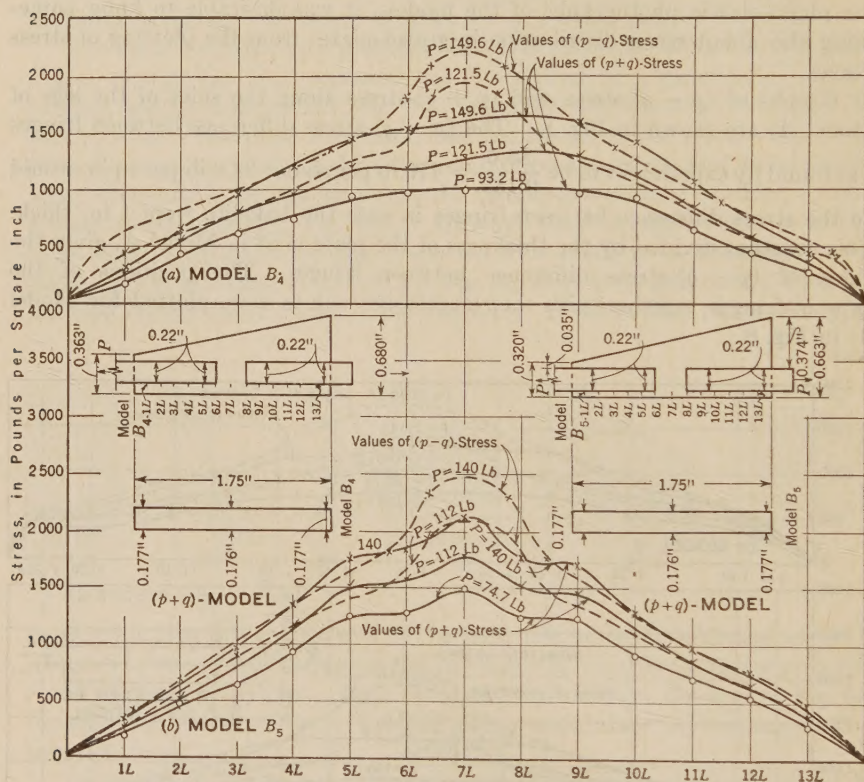


FIG. 9.— $(p - q)$ AND $(p + q)$ STRESS CURVES, MODELS B₄ AND B₅

The steepness of the curves at the ends of the plate shows the rapid building up of the stress in the plate at that point. The curves tend to flatten out near the mid-length of the legs, and from there on the $(p - q)$ -curves build up again at the ends of the legs.

The secondary humps in Fig. 9 for Model B₅ at Point 9L are due to the stress (particularly the q -stress) tending to distribute itself over a greater part of the plate. There is considerable q principal stress between the ends of the legs in the plate; its tendency to spread over a greater part of the plate depresses the $(p + q)$ -curves. No increment of $(p + q)$ -stress is added in proceeding from Point 9L to Point 8L in Model B₅. The same phenomenon of $(p + q)$ -stress

distribution is shown also between Points 5L and 6L in the tendency of the $(p + q)$ -curves to flatten out.

Sets of curves for $(p - q)$ -stress and $(p + q)$ -stress were plotted for the outer boundaries, with practical agreement. Since there is only one principal stress along these boundaries—the p -stress—the two corresponding curves for a particular model should agree. It was impossible to place the points of contact of the light interferometer exactly on the boundary. The contact points had to be moved in $\frac{1}{16}$ in. to prevent them from slipping off the model. This accounted for a slight difference in a comparison of a $(p + q)$ -curve with a $(p - q)$ -curve, which latter curve was plotted from fringe readings on the boundary. Since, for the most part, the p -stress has been the dominating stress, its direction is approximately parallel to the length of the model for most of the plate. The directions of principal stresses obtained from isoclinics, therefore, are not of great consequence for these tests.

THEORETICAL CONSIDERATION OF SYMMETRICAL GUSSET-PLATES

Many attempts have been made to find the theoretical stress distribution in symmetrical, riveted, gusset-plates, notably by the German and British investigators. Of these, one of the most recent and probably the most successful is C. Batho,⁴ who assumes a uniform distribution of stress across the sections of the plates. This assumption may be in error as much as 20%, but the error of strain energy due to this uneven distribution is only a fraction of 1 per cent. Batho set up an equation for the energy in the entire symmetrical joint which adds the energy in the plate to that of the connecting members at the plate, and the energy in the rivets. The force on each rivet or row of rivets, perpendicular to the length of the member, is taken as unknown. Derivatives are taken of the resulting equation with respect to each variable force. To make the energy in the joint a minimum, these derived equations are equated equal to zero, and solved simultaneously for the forces at each rivet or row of rivets.

In order to verify the application of this theory to a practical case, the writer found some stress measurements between the rivets of a joint (see Fig. 10) which were made by C. Findeisen.⁵ The full lines in the diagrams were plotted from Findeisen's measurements; the dotted graphs were plotted from results obtained by the application of Batho's theory by the writer to the joint tested by Findeisen. A fairly close agreement between the practical measurements and theoretical results is shown.

In a further effort to verify available theory for a symmetrical welded joint, an application of the formula for the distribution of shearing stresses in welded connections introduced by W. Hovgaard,⁶ was made by the writer to the symmetrical bakelite model, A₁. Hovgaard's theory is based upon the principle of least energy, as is the theory of Batho for symmetrical riveted joints. Instead of deriving the work equation with respect to each force, X_n , on each row of

⁴ "Riveted and Bolted Connections," by C. Batho, First Rept. of Steel Structures Research Committee, pp. 114-127.

⁵ "Versuche über die Beanspruchung in den Laschen eines gestossenen Flacheisens bei Verwendung zylindrischen Bolzen," von C. Findeisen, Verein deutscher Ingenieure, *Forschungsarbeiten*, Heft 229, Berlin, Germany.

⁶ "A New Theory of the Distribution of Shearing Stresses in Riveted and Welded Connections," by W. Hovgaard, *Transactions, Inst. of Naval Archts.*, March 26, 1931, pp. 108-126.

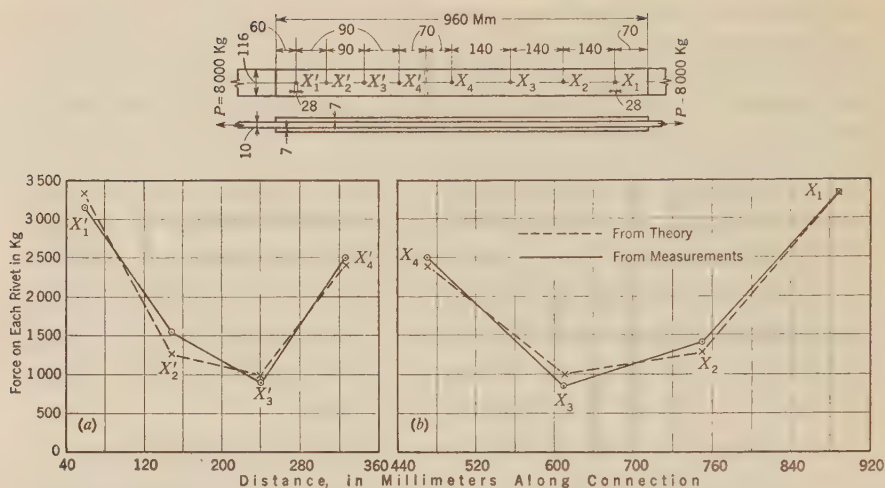
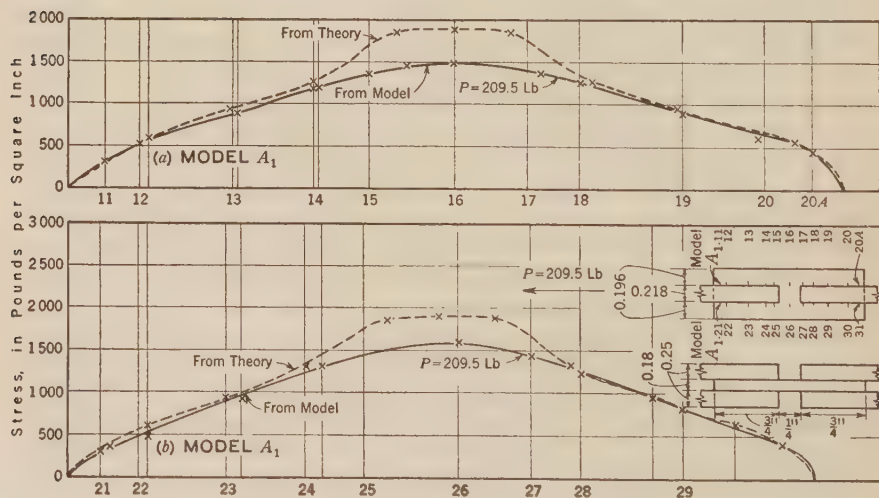


FIG. 10.—BATHO'S THEORY APPLIED TO FINDEISEN'S STRAIN MEASUREMENTS

rivets, and equating the derivatives equal to zero for a solution of least energy as Batho has done, Hovgaard writes an equation for the energy in the connection as a function of the shear stress at any point, X , of a welded connection of the member to the plate. The equation is minimized by variational calculus, and the solution of the differential equation obtained. The constants in the equation can be evaluated for a particular practical problem.

FIG. 11.—HOVGARÐ'S THEORY APPLIED TO MODEL A₁

The results of Hovgaard's theory applied to Model A₁ are shown on the plotted graphs of Fig. 11. The full lines show the stresses along the legs in the plate of the model; the stresses from the equation applied to the model are

shown by the dotted lines. From the ends of the plate for three-fourths of the distance along the legs, the curves agree very closely. Over this distance in the plate, one principal stress is much greater than the other principal stress. In fact, the smaller principal stress can be practically disregarded for this distance. The resultant stress is nearly equal to pure tension in the plate.

From the three-quarter point to the ends of the legs, the experimental curve leaves the theoretical curve for two reasons: (1) The smaller principal stress shown by experiment in Fig. 8 becomes greater, which indicates that the larger principal stress does not correspond to the exact tensile stress in the plate; and (2) there is a large increment of stress that extends from the end of the leg almost directly to the end of the other leg through the plate. Undoubtedly, the

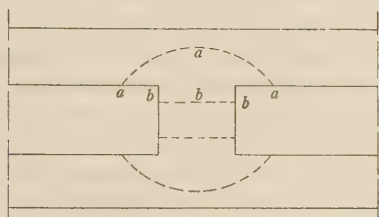


FIG. 12

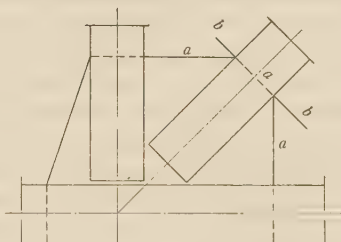


FIG. 13

second is the most important reason for the deviation of the experimental curve from the theoretical. Path $b-b$ shown in Fig. 12 is shorter and more direct than Path $a-a$; therefore, a definite increment of stress transfer takes place from the end of one leg directly to the end of the other leg. This increment relieves the transfer of stress from the side of the leg near the end, which is clearly shown by the deviation of the experimental curve from the theoretical one in Fig. 11.

RECOMMENDATIONS FOR THE SPECIFICATIONS AND DESIGN OF STEEL GUSSET-PLATES

There is a definite trend to-day toward the construction of larger steel structures with correspondingly larger gusset-plates. Sometimes, welds instead of rivets are used, but welds have not yet demonstrated completely that they are as reliable as rivets. Since the specification and design of gusset-plates have been the result largely of experience, general practise, and intuition on the part of the designer, an effort will be made to clarify and to extend the application of these known principles, as well as to add others based on the results of recent research.

A generalized stress solution for gusset-plates has not been found. In its present stage of development the Airy stress function does not offer much hope for the stress solution of gusset-plates. Likewise, there is no generalized stress solution about the phenomena of fatigue. Solutions for stress in symmetrical gusset-plates have been found both for riveted and for welded connections. The solution for symmetrical, riveted gusset-plates was found by Batho⁴ whose solution is based on the principle of least energy, as is also that

rivet around this shoulder of the hole-wall. In the fabrication of parts, improperly matched holes are not justified.

If an important gusset connection can be specified with plates of ordinary thickness and rivets of usual size, it is best to do so. The men who do the mechanical work are able to fabricate the materials of customary size with greater skill and achieve better results. Furthermore, most of the tools for fabrication of the parts are designed for average steel sections and rivets. If exceptional steel sections and rivets are specified, increments of improvement from skilled craftsmanship may be more difficult if not impossible to obtain.

Stresses.—The stresses in a plate with a re-entrant corner such as Point *A*, Fig. 14, are always greater than the elastic limit even with small forces acting on the gusset.¹⁰ Sufficient material at Point *A* changes into the plastic state

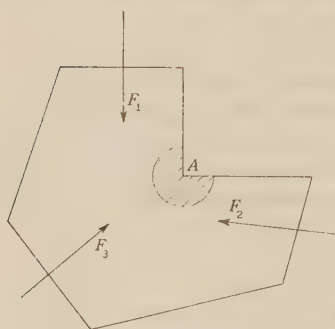


FIG. 14

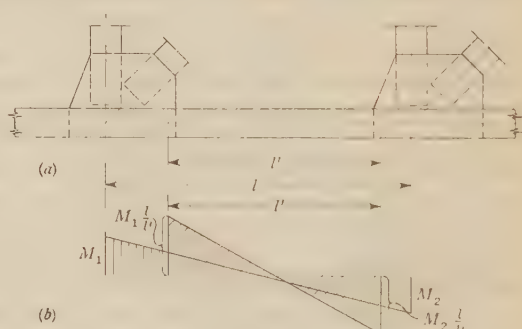


FIG. 15

to provide a means of stress transfer around the re-entrant corner. If there happens to be reversal of stress, failure occurs at relatively low forces. From the standpoint of excessive stress, gusset-plates with re-entrant cuts should never be specified. If the highly undesirable, and seldom specified, case of a gusset-plate with a re-entrant corner is encountered, a fillet in the plate at the re-entrant corner, with as large a radius as practicable, will facilitate the stress transfer. It is nearly always possible to avoid gusset-plates with re-entrant corners, but the discontinuities in stress transfer at the rivets in gusset-plates cannot be avoided.

Since the end rivets of a riveted connection are always the most highly stressed, it is of interest to consider their general method of stress transfer. If the section of the connecting member, carrying pure tension or compression, is equal to the section of the gusset, the percentage of total force taken by the end rivets varies in a manner similar to that shown in Fig. 10. If there are two rivets in a line the end rivets will take 50% of the forces. This percentage decreases to 36.6% for five rivets in a line.¹¹ From there on, for any addition to the number of rivets, the percentage of total force carried by the end rivet remains at about 36 per cent. Batho's data show that for the case in which the

¹⁰ "Stress Distribution in a Re-entrant Corner," by J. H. A. Brahtz, A. P. M. 55-6, *Transactions*, A. S. M. E., 1933, Vol. 55, pp. 31-37.

¹¹ "Riveted and Bolted Connections," by C. Batho, First Rept. of the Steel Structures Research Committee, pp. 100-136.

connecting member and the gusset-plate are equal in cross-section area, the end rivets cannot take less than 36% of the force on the member when the stress on them remains within the elastic limit. If there is no reversal, stressing the rivets beyond the elastic limit is of little consequence.

When reversal occurs, and each stress passes beyond the slip-resistance capacity of the end rivets, the ultimate strength of the connection is greatly reduced. In the design of a riveted connection for reversal of stress, one of the stresses should be kept well within the frictional capacity of the rivets with special reference to the end rivets, to prevent the backward and forward motion in the rivet holes with resultant deterioration of the connection. The cooling of the rivet and the hole-wall after driving causes a lack of tightness, which can never be avoided with heated rivets.

It may be possible to design an important connection with rivets in double shear. This procedure will double the slip capacity of the rivets as well as the shear per rivet, and might justify the additional expense in the shop and in the field. This recommendation is given added weight by the fact that at a point on the surface of the plate into which the stress is transferred in single shear, the stress in the plate is twice the average across the section of the plate, with the stress on the far surface decreasing practically to zero. In order to keep one reversible stress within the friction capacity of the joint, it may be well to re-proportion the joint or even the structure.

Even for members without reversals, there should be as many end rivets in the connection of a member as possible; or, in other words, the number of lines parallel to the length of the member should be a practical maximum. The number of these lines may be increased by welding comparatively long lug angles to the sides of the member, thus adding two or even four lines of rivets to the connection. Since a welded connection is much stiffer than a riveted one, the objection to lug angles is largely overcome. No alternate rivets in the end rows of the connection should be omitted. W. M. Wilson, M. Am. Soc. C. E., with Messrs. J. Mather and C. O. Harris, reports¹² that the section of a tensile connection is not weakened nearly as much by specifying solid end rows of rivets as is generally believed; or, conversely, the section of the member is strengthened only a few per cent by omitting alternate rivets of the end rows.

The decrease in the number of rows shortens the length of the gusset-plate, and thus limits the discontinuity that it represents. Smaller gussets also increase the clear length between joints as shown by Fig. 15(a). Moments M_1 and M_2 are from secondary effects. When l' becomes greater Moments M_1 and M_2 become less as a result of the greater slenderness ratio of the member. A. Ostenfeld¹³ states that the moments in the members at the gussets as shown in Fig. 15(b) have been increased in the ratio, $\frac{l}{l'}$, over what they would have been if the member had been free to bend through the entire distance, l . This factor, $\frac{l}{l'}$, is sometimes termed the gusset factor. In so far as the secondary stresses in a member are concerned, it is important to keep this factor as near

¹² "Tests of Joints in Wide Plates," *Bulletin No. 239*, Univ. of Illinois, Urbana, Ill.

¹³ "Teknisk Statik," A. Ostenfeld, II, p. 401, 1925.

unity as possible. From research by K. Havemann¹⁴ the length, l' , was found to extend somewhat into the gusset-plates, and he gives the formulas (see Fig. 16):

$$y = a c^x \dots \dots \dots (1)$$

and,

$$\log c = \frac{I_M}{I_G} \left(\frac{l'}{a} \left(\frac{I_G}{I_M} \right)^{\frac{a}{l'}} - \frac{a}{l'} \right) \dots \dots \dots (2)$$

in which a = distance from the old origin to the new X -axis, or the distance from the intersection of members to the end of the gusset; y = y -ordinate from the new X -axis, or the distance into the gusset from its end for l' ; I_G = effective

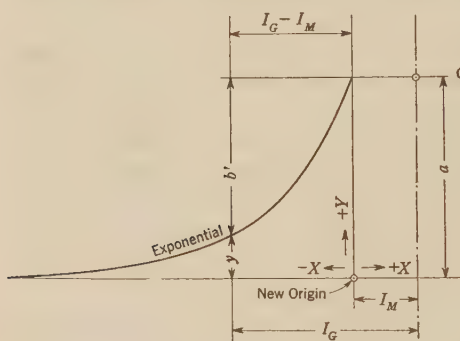


FIG. 16

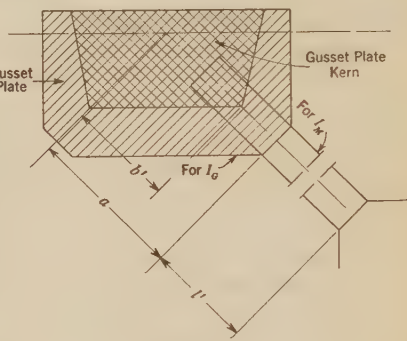


FIG. 17

moment of inertia of the gusset-plate; I_M = moment of inertia of the member; and, $b' = a - y$. The end of the gusset denotes the end of the welds or the end row of rivets. Fig. 17 shows a gusset connection with distances, a , b' , and l' of Havemann's formula.

Smaller gussets and lighter sections in the members of the trusses resulted from the multiplication of trusses in the bascule span of the Link Bridge recently completed on the Outer Drive in Chicago, Ill. The greater number of trusses with smaller gussets brought about greater lengths, l' , between the gussets for secondary bending in the members (see Fig. 15). The lighter sections also increased the $\frac{l}{r}$ ratios of the members, with a corresponding decrease in secondary bending of the members at the gussets. The smaller gussets in area and section with their smaller rivet groups are better in this bridge than the larger gussets of heavier, fewer trusses would have been.

The secondary moments in the members at the gusset-plates are usually not large enough to be of great importance in the gusset-plate stresses, but with the shortening of the rivet groups the stresses in the end rivets due to these moments should be investigated. The shorter lever arms of the end rivets about the center of gravity of a shorter rivet group increase the load on the end rivets for a given moment. These end rivets are already the most highly loaded ones from the direct stress in the member. On a section of the plate that does not

¹⁴ "Beitrag zur Untersuchung ebener Fachwerke mit steifen Knotenpunkt Verbindungen," von K. Havemann, Darmstadt, 1928.

cut any of the members, the bending stress can be found roughly by the usual theory of flexure. This case is seldom met in practice, and may not occur at the critical section. Many gussets have been designed by cutting sections and examining the direct stress, moment, and shear, considering the rivets as uniformly stressed. The bending moment by this method is found by multiplying the forces of the members by their distances from the neutral axis of the section. This moment, divided by the section modulus of the plate at the section, and multiplied by the secant² of the angle made by the edge of the plate with the normal to the section, gives an estimation of the stress from bending, at the edge of the plate.¹⁵ This procedure does not consider the non-uniform stresses in the rivets of connections as shown in Fig. 10, or the minimizing stress function shown graphically in Fig. 6(c).

The model stress data all show the non-uniform stress transfer into the plates along the legs of the models, with the greatest shearing stress transfer at the ends of the plates and the ends of the legs. Fig. 3 also shows the tendency for concentration of stress in the direct path between the model legs from the principle of least energy. The highest ($p - q$)-fringes shift toward the narrow side of the plate in the unsymmetrical plates, or in the case of Model D_3 toward the side of the plate with least area. If the unsupported edge of a steel gusset-plate is stressed in compression, the edge will buckle before failure, throwing more moment and direct stress into the interior of the plate. Parts of the gusset-plate in direct line between members should be designed with the object of providing for all these additional stress increments. The connections of the members to the plate should be made wide, with as many lines of rivets for a riveted connection as practicable.

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¹⁵ "Bridge Engineering," by J. A. L. Waddell, 1925, pp. 519-529.

¹⁶ This paper is a part of a thesis submitted by the writer for a D.Sc. degree at the Univ. of Michigan. A complete copy of the thesis has been filed for reference in Engineering Societies Library, 29 West 39th Street, New York, N. Y.

